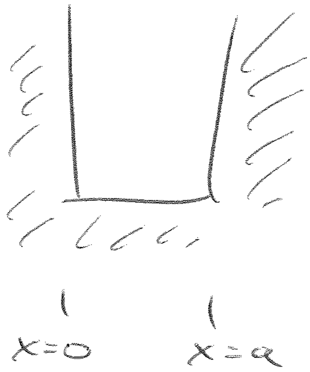
Quantum Physics I - Nov. 2017

1]



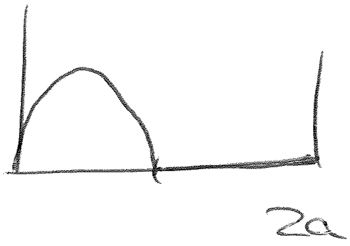
a) eigenstates: $\sin\left(\frac{n\pi x}{a}\right)$ $n=1,2,\ldots$

energies: $E = \frac{n^2\pi^2\hbar^2}{2\cdot m\cdot a^2}$ quantization from $\psi(x=a)=0$

b) NOT a stationary state
since a general symm. function is a linear combination of eigenstates with n odd

c) ANY outcome E_n $\forall n$ is possible.

d)



not symm. / anti-symm

$\Rightarrow$ again ANY outcome

2] a) compatible $\simeq$ commuting operators
$\Rightarrow$ can be diagonalised at the same time = have the same basis of eigen-states.

b) $[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$ $\Rightarrow$ incompatible.

c) again, $\vec{p}$ and $\vec{L}$ do not commute
$\Rightarrow$ incomp.

3] a) entangled state cannot be written as a product of two one-particle states (also after a basis change).

e.g. $|\uparrow\uparrow\rangle$ is not entangled.

b) $\frac{1}{\sqrt{2}} \cdot (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ is entangled.
We showed in the exercises that you cannot rewrite this as a product state:
$(\quad)_{\text{first}} \cdot (\quad)_{\text{second fermion}}$.

c) Measurement of e^- $\Rightarrow$ that of e^+.
But both are random - but correlated.
So QT is non-local. You cannot send a signal however $\Rightarrow$ still causal.

4] $\psi = R \cdot \Theta \qquad R(r) = \frac{2}{a^{3/2}} e^{-r/a}$

a) groundstate = spher. symm. ⇒ no dependence on angles

b) $H\psi = E\psi$

contains /

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2 = -\frac{\hbar^2}{2m} \cdot \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) \text{ on } \psi$$

$$\Rightarrow \cancel{H\psi} = -\frac{\hbar^2}{2m} \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^2 \cdot -\frac{1}{a} \cdot e^{-r/a}\right]$$

$$= -\frac{\hbar^2}{2m} \cdot \left(\frac{1}{a^2} \cdot e^{-r/a} + -\frac{2a}{r} \cdot e^{-r/a}\right)$$

only part that talks to RHS

$$\Rightarrow E = -\frac{\hbar^2}{2m} \cdot \frac{1}{a^2}$$

(note: normalisation not important here)

wrong factors -1 per factor

Not separate ε, V = -2 pt.

c) $H = T + V$ ② → Put it in 3pt.

Wrong units -2 pt! +1 pt it realise.

⇒ in order to satisfy Schr. eq., you need

$$V = -\frac{\hbar^2 a}{m r^2}$$

R A: upl.

d) $$\langle r \rangle = \int_0^\infty R^* \cdot R \cdot R \cdot R^2 \, dR$$

$$= \frac{4}{a^3} \int e^{-2R/a} \cdot R^3 \, dR \qquad R = x/2$$

$$= \frac{4}{a^3} \int e^{-x/a} \cdot \left(\frac{x}{2}\right)^3 \cdot \frac{dx}{2}$$

$$= \frac{1}{4 \cdot a^3} \cdot 3! \cdot a^{4} = \frac{3}{2} \cdot a$$

$-n=2$

Forgot $\ell=0$ → -1   Dont listing -1 $n=2$: 1pt

e) first excited state: $n=2,$ $\ell=1,$ $m_\ell = 0, \pm 1.$
1
$\ell = 0,$ $m_\ell = 0.$
2   2

f) $\langle r \rangle$ – smallest for ground state
– larger for $n=2,$ $\ell=0$
– largest for $n=2,$ $\ell=\cancel{2} 1.$

$n \uparrow \langle r \rangle \uparrow$ ⇒ 3pt
$\ell \uparrow \langle r \rangle \uparrow$ ⇒

1 correct 3pt